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## MODELING RADAR CLUTTER

David A. Sparrow

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David A. Sparrow

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## PREFACE

The goal of this work is to improve modeling of the engagement of aircraft by radar systems. To keep this paper unclassified, we treat engagements characterized by an arbitrarily chosen 50 percent probability of detection at 10 km, and excursions therefrom. No attempt has been made to link this performance to any combination of target signatures and radar performance. Nevertheless, this document should be treated as sensitive.



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## ABBREVIATIONS

ADC      Analog to Digital Computer

DMA      Defense Mapping Agency

MTI      Moving Target Indicator

$P_D$       Probability of Detection

PRF      Pulse Repetition Frequency

SCAT      scout or attack (helicopter)

S/C      Signal to Clutter

S/I      Signal to Interference

S/N      Signal to Noise

## EXECUTIVE SUMMARY

Many analyses of the engagement of low-flying targets by radar-aided weapons treat radar clutter by using a single characteristic reflectivity value. This reflectivity is used to compute a clutter return, as a function of range to target, which is added to the system noise to obtain a total interference. This total interference is then used in standard formulae for noise-limited detection. Rarely is this established methodology likely to be accurate where clutter is important.

The intensity of environmental radar returns varies greatly, and this variability is one of the dominant features of the engagement of low-flying targets by radars. The broad distribution of radar returns from environmental features leads to clutter-limited detection probabilities that approach unity slowly as range decreases, rather than abruptly as in the noise-limited case. The probabilities of successful engagement at short ranges are greatly influenced by clutter reflectivities far from the median value. Whether or not a target is detected on a particular scan depends in large measure on the clutter return from the target's range cell. The distribution of clutter reflectivities is often so wide that variations of many decibels in signature or threshold of detection correspond to changes of only a few percent in probability of detection ( $P_D$ ). Thus, where clutter variability is large, it must be included to avoid errors. However, large clutter variability when included will tend to overwhelm the uncertainties from other sources such as human performance. This may allow simplified treatment of these other sources.

This study focused on engagement of helicopters, and treated both steady signatures characteristic of return from an aircraft body and the highly modulated signatures characteristic of hub/rotor returns. The results on body signatures apply as well to the modeling of engagements with low-flying, fixed-wing aircraft and cruise missiles.

## I. INTRODUCTION

A recently released IDA study entitled "Active and Passive Aids to Survivability for LHX" (Ref. 1) investigated signature reduction as a survivability aid for combat helicopters. One of this study's conclusions was that the variability in anticipated environmental clutter levels was so great that traditional analyses using a single (typically median) value were untrustworthy. The main purpose of the work reported here is to outline how to incorporate a wide range of clutter values in combat models, and to indicate how this would influence effectiveness studies.

The study of radar detection of aircraft has focused primarily on fixed-wing rather than rotary-wing targets. Fixed-wing aircraft targets move at a given velocity with a fairly steady signature. Detection of high-altitude aircraft is limited primarily by radar system noise, rather than obscured by competing returns from ground clutter. This feature does not extend to scout or attack (SCAT) helicopters employed according to current Army doctrine.

SCAT helicopters survive in combat primarily through the use of terrain masking to screen themselves from enemies. This places them well within a radar beam width of terrain features, and the return from these features (ground clutter) can often protect a low-flying helicopter from radar systems even when it is exposed. At the short ranges characteristic of exposure and engagement of low-flying helicopters this clutter is much more likely to afford protection than system noise.

The profile of helicopter signatures in velocity and time is also very different from that of fixed-wing aircraft. One of the largest signatures is the blade flash, the specular return from the entire length of the blade as it comes broadside to the radar. This return is of very short duration, and is repeated with the blade passage frequency, or possibly twice that frequency if there is sufficient lead/lag between opposing blades or if the number of blades is odd. The velocity spectrum of this component of the signature covers most of the region from Mach -1 to +1. On many helicopters there is also an appreciable return from rotor-body interactions which are also intermittent, and extend over a wide range of velocities but are harder to characterize simply.

The body signature of a helicopter has variation comparable with that of fixed-wing aircraft, but the aircraft velocities are much slower. This is important because the primary electronic clutter suppression techniques exploit target velocity. The total radar return from a low-altitude helicopter is likely to be overwhelmed in magnitude by the return from clutter. If the helicopter is moving fairly fast, the radar return from the body will have a large enough Doppler shift of its transmittal frequency to be distinguishable from the unshifted return from the clutter. Even for a hovering helicopter the signature involving the moving parts is, in principle, distinguishable from the clutter, but is likely to be only infrequently large enough to detect.

Experience with high-flying, fixed-wing aircraft leads to analysis in terms of an engagement range, inside of which the aircraft is vulnerable. For a given radar system there is a fixed noise level, the signal-to-noise (S/N) ratio falls very fast as a function of range ( $1/R^4$ ), and the probability of detection (subject to a given false-alarm rate) as a function of signal-to-noise ratio switches fairly rapidly from near 100 percent to near zero in the neighborhood of a critical value of S/N. As a result, the detection process can be characterized by a critical range (see Fig. 1 for typical profiles of probability of detection versus range. These curves will be discussed in some detail in Chapter II).

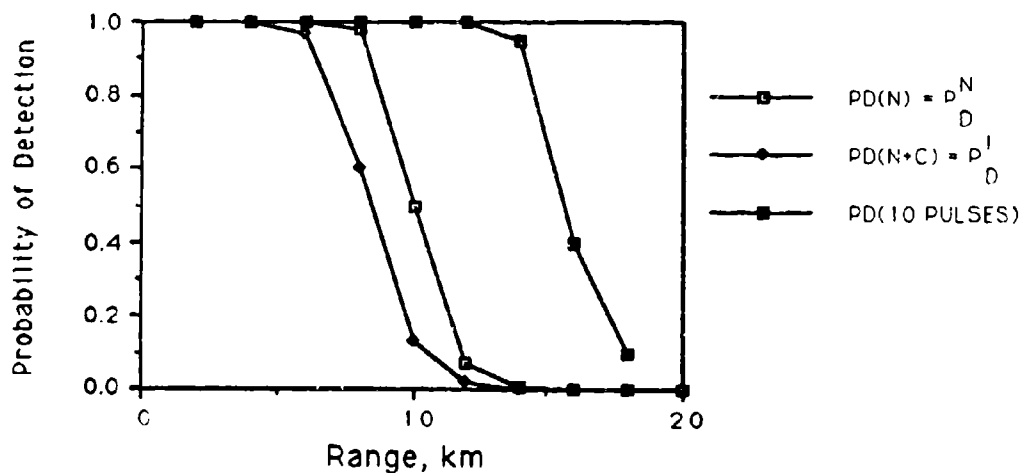


Figure 1. Noise-Limited Detection

Tracking and other systems generally are engineered to be better than the acquisition radar. Hence, using the 90 percent or 50 percent probability of detection range for the

acquisition radar as an engagement range is a reasonable practice. Any time delays in the engagement result from hand-over times.

To a large extent this fixed-wing engagement background has influenced the modeling of engagements of low-flying helicopters by radar air defense units. The parallel breaks down in several particulars. First, clutter rather than noise limits detection, and these environmental clutter returns show a broad distribution of values at any given location, and considerable variation in the distributions from place to place. Second, even for a fixed value of clutter reflectivity the signal-to-clutter ratio (S/C) falls much more slowly with increasing range ( $1/R$ ) than S/N. In addition, the probability of detection in heavy clutter is more influenced by human factors than is noise-limited detection, and as a result is much less predictable. Finally, for hovering or very slow moving helicopters, timelines for engagement may be dominated by the intermittent nature of the exploitable signatures.

To add to the complexity, the helicopter can be masked or exposed; the clutter in the same range gate as the target can be masked or exposed; if the helicopter is visually masked the possibility exists that diffraction of the radar beam will allow detection, and if the helicopter is exposed there may be multipath signals bouncing off the terrain that limit detection or that introduce tracking errors. Furthermore, windblown foliage and rain can introduce a time dependence to the processed clutter.

In order to focus this study on achievable goals we will treat the impact of broad clutter distributions on helicopter detection, both for the case of steady aircraft signature, characteristic of returns from the body in fast forward flight, and for the case of time varying signature, characteristic of the returns involving moving parts that dominate the exploitable signature of a helicopter in hover (Chapters II and III, respectively). In a model or war game that includes a specific terrain map we recommend determining if the ground clutter is masked, using the same algorithm in use to determine if the target was masked. Clutter distributions could be incorporated easily, relatively, if at every point one drew from the world clutter distribution. A more realistic, and tactically more relevant, approach would be to draw from distributions appropriate to local features including terrain type, ground cover, and grazing angle. In the fourth section we propose an implementation plan to include a broad clutter spectrum in Army models of detection of hovering and transiting helicopters. The expected impacts of these modeling improvements are discussed in Chapter V. The appendix contains some derivations and examples concerning engagement timelines of requiring m-out-of-n detections to declare a target.

## II. TARGETS WITH A STEADY SIGNATURE

As a practical matter, detection of a target in clutter depends on a complicated array of factors involving the radar, the operator, and the environment, as well as the target signature. The challenge we face is to describe the impact of clutter on engagement in a fashion suitable both for one-on-one analyses and for more intricate war games. The complexity of the problem motivates a search for a dominant feature around which the analysis can be organized.

The initial step in an engagement analysis is to determine probability of detection. The probability is determined by a number of features all of which are uncertain or variable. The variability in environmental radar returns dominates all other variabilities and uncertainties. Hence, it is this variability, rather than any average behavior of this or other features of the detection, which will be central to the analysis.

Before presenting the formalism for including clutter variability, we will qualitatively support the assertion of the dominance of clutter variability by comparing a number of possible changes to a radar or a target. For single pulse detection, a change in probability of false alarm from  $10^{-2}$  to  $10^{-12}$  changes the S/N requirement by 7 dB for 90 percent  $P_D$  and 8 dB for 50 percent  $P_D$ . Increasing the number of pulses on target from one to ten decreases the S/N requirement by around 8 dB. For the radar to double linear dimension could lead to a 12 dB improvement in S/N with a 3 dB improvement in S/C. Turning to clutter suppression, if the analog to digital converter dynamic range is limiting clutter suppression, increasing the number of bits by 2, say from an 8-bit to 10-bit analog to digital converter (ADC) would improve clutter suppression by at most 12 dB. A technology change from 2-pulse to 3-pulse cancellers in an MTI system improves theoretical clutter suppression by around 20 dB for typical values of PRF and dwell time. In contrast, clutter reflectivity values in a single location are characteristically spread over more than 30 dB.

For a given processed signal to clutter ratio, S/C, at the target pixel the probability of detection will depend on operator skill, the clutter present in nearby pixels, if not the whole screen, and the presence of moving target cues. S/C itself depends on the target's

signature, the radar's clutter suppression capability, and the local environmental clutter reflectivity:

$$S/C = \frac{\sigma I_{SC}(v)}{\sigma_o R \theta r} \quad (1)$$

where  $\sigma$  = target signature

$\sigma_o$  = clutter reflectivity

$R$  = range to target

$\theta$  = azimuthal beam width

$r$  = range resolution

$I_{SC}(v)$  = clutter improvement capability, which depends on radial velocity,  $v$ .

The probability of detection over a particular spot depends on the signal to clutter ratio,  $S/C$ , and a set of other variables  $\{\alpha\}$ , which include specifics of target radar cross section, human factors, the surrounding environment, etc. This probability has hidden dependence on range through  $S/C$ , and may be averaged over the distribution of clutter reflectivities in order to obtain a probability as a function of range characteristic of this distribution of clutter reflectivities rather than a single value of the clutter reflectivity.

$$P_D^C(R, \{\alpha\}) = \int d\sigma_o P_D(S/C, \{\alpha\}) \rho(\sigma_o) \quad (2)$$

If the clutter spectrum  $\rho(\sigma_o)$  is broad then the probability of detection for a given  $S/C$ ,  $P_D(S/C, \{\alpha\})$ , can be replaced by a threshold, the exact value of which depends upon  $\{\alpha\}$ :

$$P_D^C(R) = \int_{-\infty}^{T_C} d\sigma_o \rho(\sigma_o) = P_{cum}(T_C) \quad (3)$$

where:

$$T_C = \sigma I_{SC}(v)/T R \theta r$$

and  $T = S/C$  required at the threshold, and the probability of detection is the probability that the clutter reflectivity is below some definite value  $T_C$ . The  $-\infty$  in the lower limit of Eq. 3 presumes a dB scale for  $\sigma_0$ .

We note that the distribution of clutter reflectivities is often so wide that variations of many dB in signature or threshold of detection correspond to changes of only a few percent in  $P_D$ . Thus, where clutter variability is large, it must be included to avoid errors. However, large clutter variability when included will tend to overwhelm the uncertainties from other sources. Mathematically, this was accomplished by replacing the probability distribution of Eq. 2 with a threshold in Eq. 3.

We will now compare the results for detection probability using a variety of models. Since the effects of clutter have generally been added to an approach focused on system noise, we begin with a review of how noise limits detection. A radar has a threshold and a gain, and under the assumption of Gaussian noise these can be used to compute how detection probability varies with S/N for a given false alarm rate. These relationships are tabulated in many standard references (cf. Refs. 2, 3).

In Fig. 1 we show noise-limited, single-pulse detection probability as a function of range,  $P_D^N(R)$  (open squares) for a radar with 50 percent  $P_D^N(R)$  at 10 km with a  $P_{fa}$  of  $10^{-6}$ . If the radar integrates over 10 pulses this detection probability would occur between 15 and 16 km (solid squares). (See Ref. 2 for plots of  $P_D$  versus required S/N for various numbers of pulses.)

One standard method for incorporating clutter is to compute a signal-to-interference (S/I) ratio  $S/I = S/(N + C)$ , and use S/I in place of S/N to determine  $P_D^I(R)$ . We have assumed a uniform clutter reflectivity such that S/C is 13 dB at 10 km. This example has been set up so that clutter and noise are comparable at 10 km. This is shown in Fig. 2 where S/I is dominated by clutter inside 8 km and by noise outside 10 km. Returning to Fig. 1, we see that computing  $P_D$  using S/I shortens the engagement envelope by a couple of kilometers.



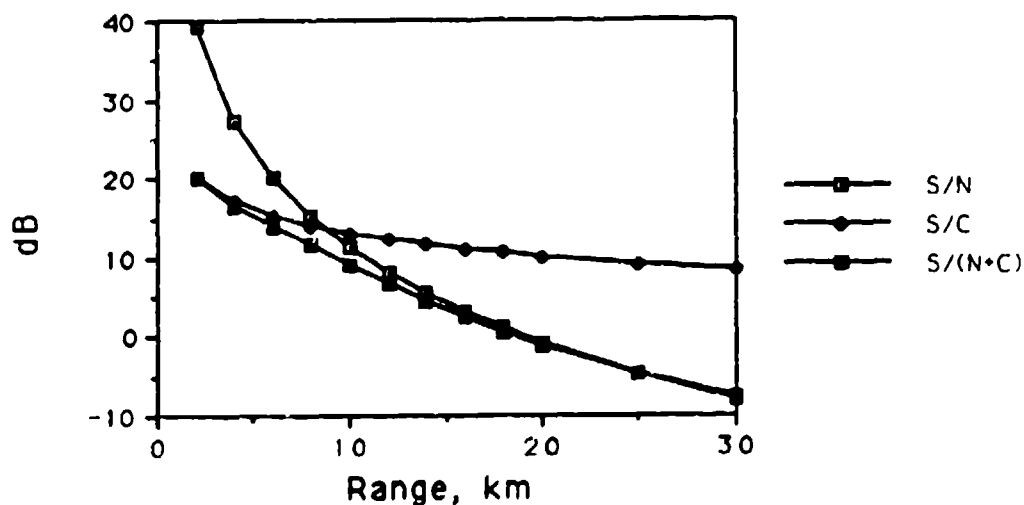
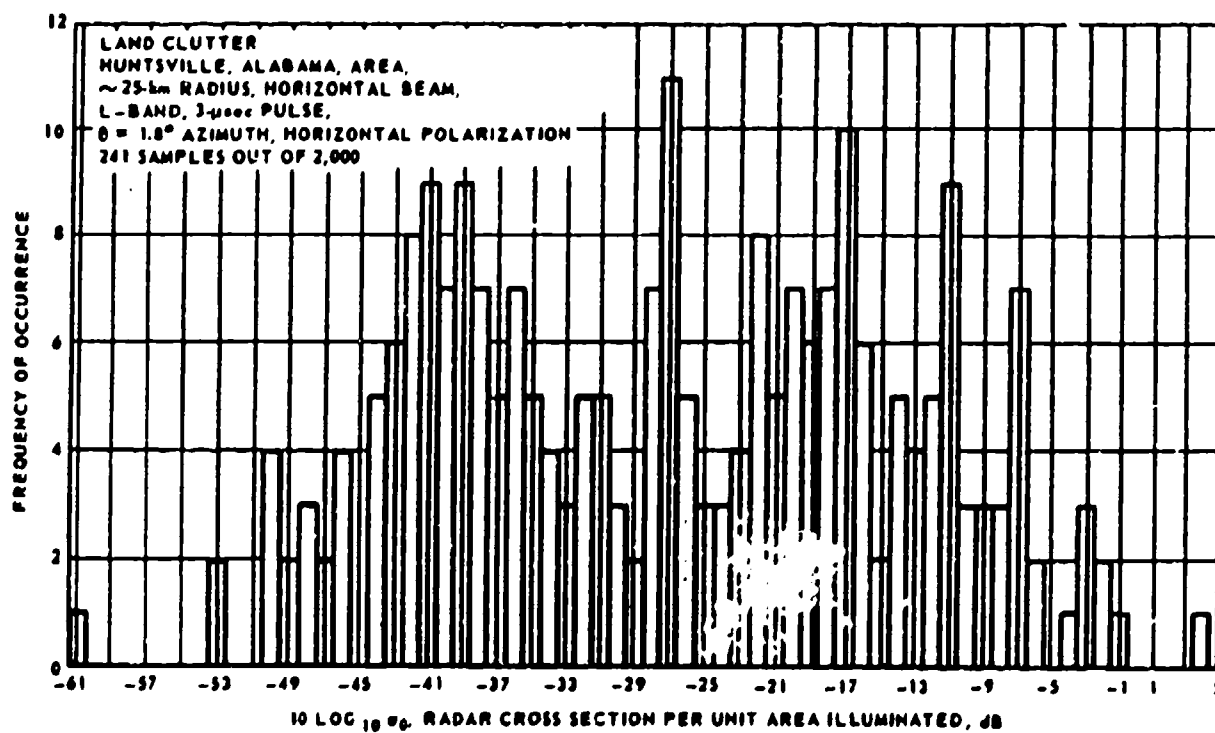


Figure 2. Signal to Interference

This approach relies on use of a single reflectivity to characterize the clutter. In order to include the range of clutter returns we have described the Huntsville area clutter reflectivity values reported in Ref. 3 (see Fig. 3) by the following representation of the cumulative probability distribution:

$$P_{\text{CUM}}(\sigma_o) = \frac{e^{(\sigma_o - \sigma_m)/\sigma_w}}{1 + e^{(\sigma_o - \sigma_m)/\sigma_w}} \quad (4)$$

Using for the median value,  $\sigma_m = -27$  dB, and for the width,  $\sigma_w = 16$  dB gives the fit presented in Fig. 4. The 10-90 percent width of the distribution is  $2 \ln(3) \sigma_w \approx 35$  dB, over three orders of magnitude. Most of the discrepancy in the fit results from using a smooth function to describe the Huntsville clutter, which is bimodal with a minimum near the median value. We will discuss below using different clutter distributions in different terrain, rather than a global distribution applied everywhere, at least in simulations with combat scenarios played out in particular terrain. For simplicity in computing generic engagement envelopes, we will use the parameterization above.



*Histogram of occurrence of σ<sub>0</sub>. (After Holliday [176].)*

Figure 3. Environmental Radar Clutter Distributions

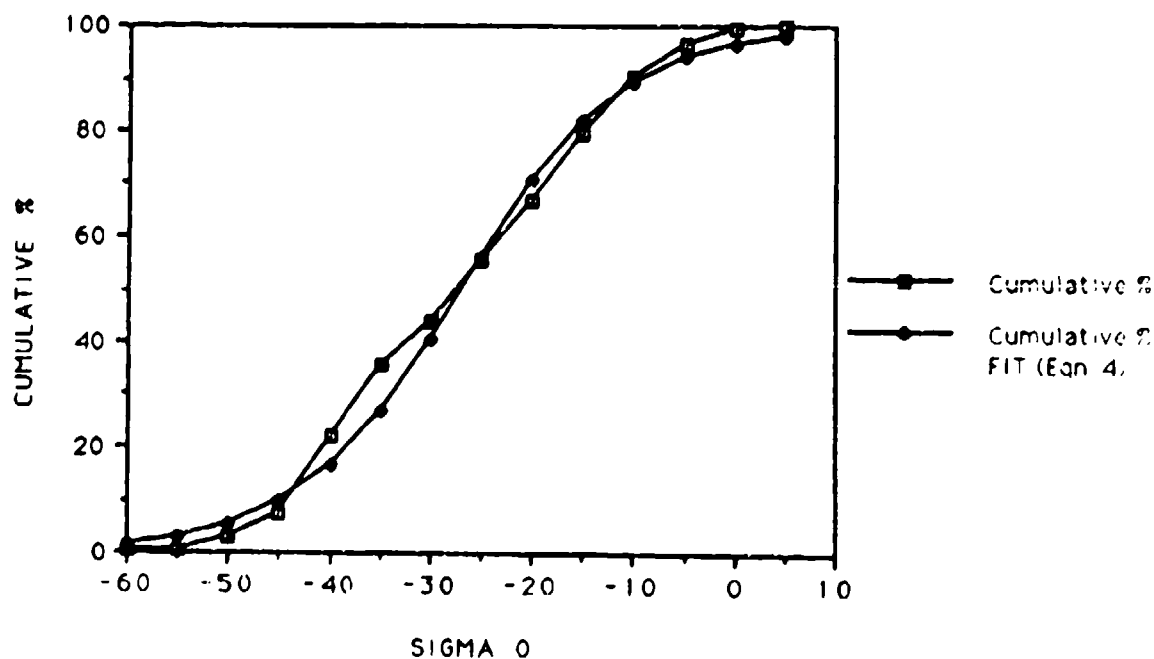


Figure 4. Comparison of Huntsville Data and Fit

To compare the approach of Eq. 3 to the S/I approach, we assume the radar parameters are such that a threshold based on an S/C requirement of 13 dB leads via Eq. 3 to a 50 percent probability of detection at 10 km for the Huntsville  $\rho(\sigma_0)$ , if the effects of noise are neglected. This is the same as would have been obtained with the signal-to-interference method if noise were neglected there also. However, as we see in Fig. 5, the curve describing the probability that  $S/C \geq 13 \text{ dB} = 20$  (solid diamonds) is very flat compared to  $P_D$  curves, whether S/N or S/I is used (open squares, or any curves from Fig. 1).

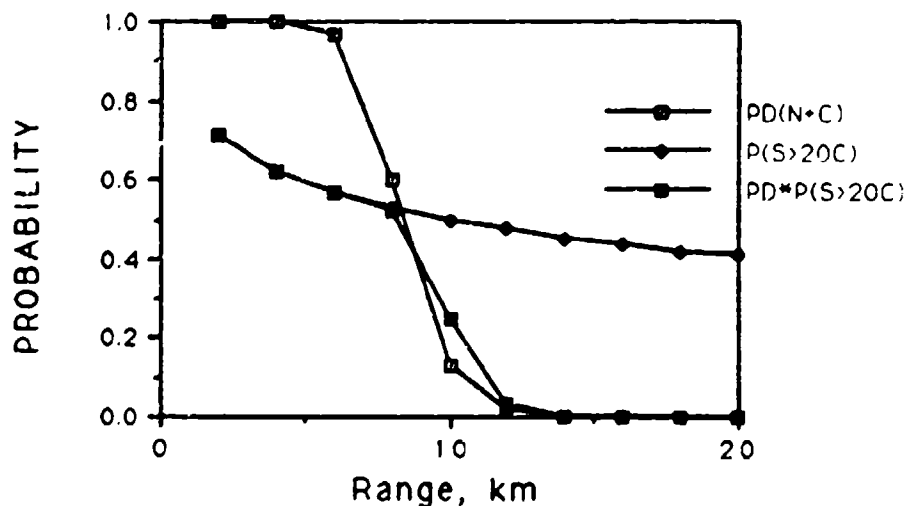


Figure 5. Clutter-Limited Detection

Neglecting noise altogether is clearly inadequate. We propose as a surrogate for the overall probability of detection limited by both noise and clutter the product of probabilities:

$$P_D(R) = P_D^C(R) P_D^N(R) , \quad (5)$$

which is represented by the solid squares. This curve is very different from the curve obtained by replacing S/N with S/(N + C) (open squares). Inclusion of the full clutter distribution shows that some protection is provided even at very short ranges. Changing the width parameter to  $\sigma_w = 6 \text{ dB}$  yields a much narrower clutter distribution, but the expected probability of escaping detection is still significant at short ranges.

Thus far we have been considering a single scan. The scan-to-scan behavior can be regarded as correlated or uncorrelated depending on whether or not there is motion from

cell to cell on the radar screen. Within this model, in the absence of motion, the cumulative probability of detection with time is identical to the single-scan probability in the clutter-dominated region. At a minimum this neglects some time dependence of human performance which is buried in  $\{\alpha\}$ . These effects are outside the scope of this work.

If the target is moving the cumulative detection probability is given by

$$P_D^{\text{cum}}(\{R_n\}, t) = 1 - \prod_n (1 - P_D(R_n)) \quad (6)$$

where  $n$  is the number of scans, the time elapsed is  $t = n\tau$ , where  $\tau$  is the scan period, and the  $\{R_n\}$  are the target locations during illumination in scan  $n$ . This probability will increase monotonically to unity since sooner or later the helicopter will appear over a patch with low clutter.

For a proper treatment, one would wish to include a clutter distribution that was location specific, including such effects as grazing angle, terrain type, and clutter masking. Provided that within these categories the clutter spectrum is still broad we can again approximate the complex problem with a threshold. The full expression for clutter-dominated probability of detection would now depend on the location of the air defense radar ( $\vec{R}_{AD}$ ) as well as the range and direction ( $\vec{R}$ ) to the target:

$$P_D^C(\vec{R}_{AD}, \vec{R}) = \int_{-\infty}^{\infty} d\sigma_o \rho(\sigma_o, \vec{R}_{AD}, \vec{R}, \{\alpha\}) \quad (7)$$

where the variables in  $\{\alpha\}$ , such as terrain type, now influence  $\rho$ , and are not simply aggregated into the threshold. Time dependence of detection would be treated as above. In this case, however, the dependence of the probability of detection on the  $\{R_n\}$  would in general include very rapid variation due to changes in terrain type or grazing angle, in addition to the relatively slow variation due to changes in range.

### III. TIME VARYING TARGETS

To treat a time varying signal a probability density function,  $\rho_T(\sigma)$ , describing the time distribution of the target signature, must be incorporated. Then, assuming the radar scan rate and the blade passage frequency are not synchronized, Eq. 2 can be modified by including an average over  $\rho_T(\sigma)$ :

$$P_D^{SS}(R) = \int d\sigma_0 \rho(\sigma_0) \int d\sigma \rho_T(\sigma) P_D(S/C, \{\alpha\}) \quad (8)$$

Note that given the nature of the returns involving the hub/rotor system, especially the blade flash,  $\rho_T(\sigma)$ , the probability density for the sampled cross section depends upon radar parameters as well as the target. For example, MTI systems will process hub and blade returns differently, depending on the pulse repetition frequency, and scan rate and beam width will determine the probability per scan that a bright, brief blade flash will be returned.

If we again assume that the broad clutter spectrum permits use of a threshold, we may simplify Eq. 8 to:

$$P_D^{SS}(R) = \int d\sigma_0 \rho(\sigma_0) \int_{T_S} d\sigma \rho_T(\sigma) \quad (9)$$

where  $T_S = T R \theta r \sigma_0 / I_{sc}(v)$ .

A very simple model of  $\rho_T(\sigma)$  would be a sum of two weighted delta functions at the values of  $\sigma$  corresponding to capturing a blade flash or not. The probability of capturing the flash would depend on the number of blades, main rotor rate, radar beam width, and radar scan period. For example, for an even-bladed system where advancing and retreating blades were captured simultaneously

$$P_T(\sigma_F) = \Delta \theta N f / \omega \delta(\sigma - \sigma_F)$$

yielding for the probability of capturing a flash

$$P_T(\sigma_F) = \Delta \theta N f / \omega$$

where

$$\Delta\theta = \text{radar beam width} \quad (10)$$

$\omega$  = scan rate

$N$  = number of blades

$f$  = rotor frequency, and

$$P_T(\sigma_{NF}) \equiv 1 - P_T(\sigma_F).$$

In such a simple case the integral over signature could be replaced with a two-term sum with different thresholds for the remaining integral over clutter reflectivity. This would give for a single-scan probability of detection for a helicopter over a randomly selected ground patch

$$P_D^{SS}(R) = P(\sigma_F) \int_{-\infty}^{T_F} d\sigma_o \rho(\sigma_o) + P(\sigma_{NF}) \int_{-\infty}^{T_{NF}} d\sigma_o \rho(\sigma_o) \quad (11)$$

where the upper limits are defined as in Eq. 3.

Generally speaking, if the helicopter is hovering or moving very slowly the time varying signature dominates the detection. Assuming the helicopter is essentially stationary over a particular ground patch with a fixed value of clutter reflectivity,  $\sigma_o$ ,

$$P_D^{SS}(R, \sigma_o) = \int_{T_s}^{\infty} d\sigma \rho(\sigma) \quad (12)$$

For a given  $\sigma_o$ , a signature distribution with just two values would reduce to either unity,  $\rho_T(\sigma_F)$ , or zero for the probability depending on whether both values, only the higher, or neither resulted in detection. In general, the cumulative probability of detection would be given by

$$P_D^{cum}(R, t, \sigma_o) = 1 - \left(1 - P_D^{SS}(R, \sigma_o)\right)^n \quad (13)$$

To compute an aggregated cumulative probability this result is averaged over the distribution of  $\sigma_o$  values:

$$P_D^{cum}(R, t) = \int_{-\infty}^{\infty} d\sigma_o \rho(\sigma_o) P_D^{cum}(R, t, \sigma_o) \quad (14)$$

We have not included system noise or location specific clutter reflectivity distributions in this discussion. These features can be included in the same fashion as in Chapter II.

At this point we would like to contrast detection of steady signatures (from a helicopter body) with detection of highly intermittent signatures (from a rub/rotor system) using a relatively high PRF, fast scan radar typical of those employed in air defense applications. We again assume 50 percent probability of detection based on a single pulse at 10 km in the absence of clutter, and 50 percent probability that the Huntsville clutter distribution would mask the target at 10 km if noise were negligible. Finally, we assume the same signature for the body and the peak blade flash, but the body to be illuminated for 10 pulses, while the flash is so narrow that only a single pulse captures the peak return.

Detection of the body is illustrated in Fig. 6, for the cases of no clutter (solid diamonds) and clutter plus noise as modeled above (solid squares). Figure 7 shows a similar set of curves for detection given that a blade flash was captured. The curves are very similar to those in Fig. 6, except that noise limits the detection to shorter range with single pulse detection than with 10 pulses.

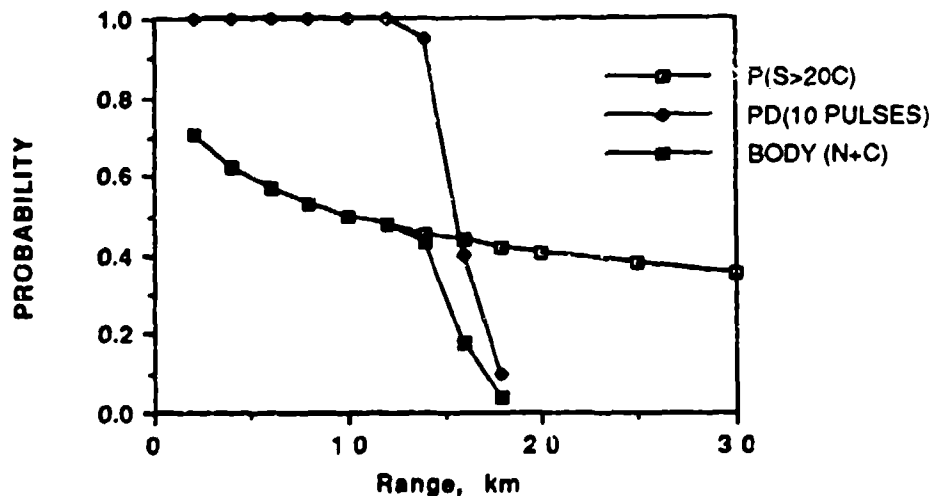


Figure 6. Detection of Helicopter Body

Probability of detection as a function of range given a flash is a misleading quantity because of the intermittent nature of the flashes. The hub/rotor return is modeled as the sum of a flash plus a background return. We assume this return was captured by the radar

at the flash level for a single pulse 10 percent of the time, and a much lower hub return 90 percent of the time. Assuming the hub return to be 10 dB lower leads to a probability of detection indicated by the open diamonds for detection by a single pulse off the blade or 10 pulses off the hub. For ease of comparison the noise plus clutter limited body and rotor detection probabilities are repeated in Fig. 8 and a case with the hub 20 dB below the rotor flash is added.

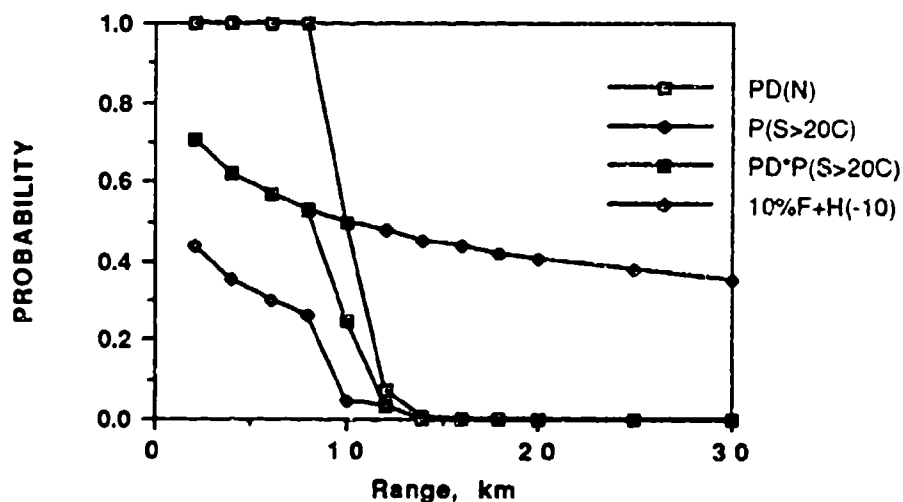


Figure 7. Detection of Blade Flash

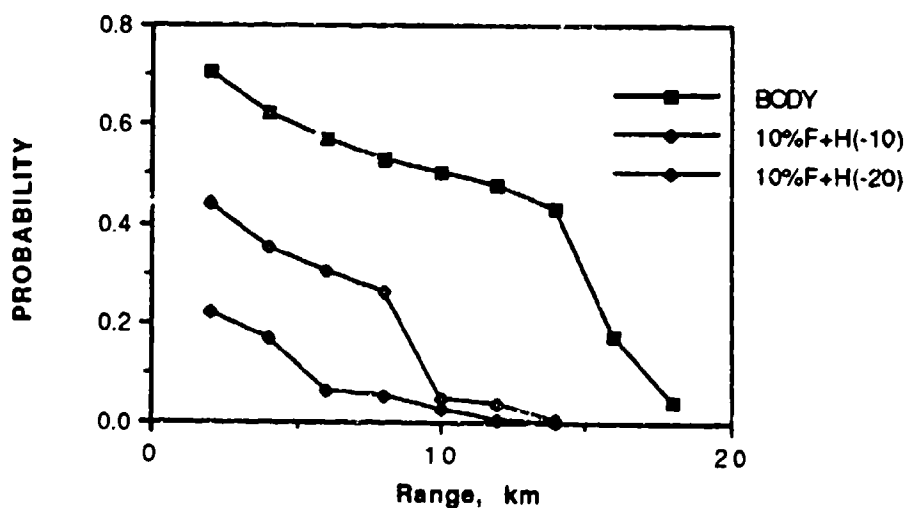


Figure 8. Comparison of Body versus Rotor Detection



## **IV. IMPROVING THE TREATMENT OF CLUTTER IN COMBAT MODELS**

Simple engagement envelopes, like those presented in Chapters II and III, are useful for qualitative comparisons and for demonstrating the importance of features to be included in more detailed analysis. These chapters show that the large width of clutter distributions is a feature which warrants inclusion in models such as JANUS and CASFOREM, at least when they are used to support acquisition decisions involving helicopters, other vehicles operating in clutter, or the radar-directed weapons that engage them. In this chapter we will discuss how to incorporate the effects of clutter in the engagement of helicopters by air defense systems.

### **A. MASKING**

Since use of terrain masking is a primary helicopter survivability tactic, models usually do not permit engagement when the helicopter is masked. This approach should be extended to set the clutter return to zero if the ground in the same range gate with the target is masked. We believe that this would be a relatively simple coding change since the line-of-sight algorithm used to determine visibility of the helicopter could also be used to determine visibility of the ground. This change would increase the detectability of targets uniformly. It is difficult, however, to assess in the abstract how important this change will be, because it is likely to be very dependent on terrain and on mission. Nevertheless, implementation should be so simple that we believe this is the number one improvement to be made in modeling effects of clutter for those codes where masking of ground clutter is not already incorporated.

### **B. USE OF A UNIVERSAL CLUTTER DISTRIBUTION**

Clutter reflectivity distributions depend on the radar's frequency, polarization and resolution cell size, as well as environmental features such as terrain type, grazing angle, man-made features, and vegetation cover. By a "universal" distribution we mean one applied to the entire globe, but which at least acknowledges the well-known dependence of clutter reflectivity on radar frequency. To sample the clutter distribution would require

some sort of random draw, but several conditions must be applied if this random draw is to make both physical and operational sense.

First, the distribution must be based on map coordinates in a way that is reproducible if vehicles leave and return to a specific location. Additionally, it would be desirable for correlations to exist among the clutter reflectivities computed at various ranges along the same azimuth from the target.

Second, as the location of the illuminated ground patch changes, the clutter reflectivity should not change much over a region small compared to a beam illumination region.

Finally, as the azimuth from the illuminated ground patch changes, the clutter reflectivity should change smoothly. We are uncertain at present of the phenomenology necessary to quantify this statement further.

There are two basic ways to implement such a scheme. The first approach would use extra fields in the map data to record clutter reflectivities with azimuthal, frequency and grazing angle dependence. If these numbers were measured for a given location so much the better. If not measured, they could be estimated, consistent with known phenomenology. The second approach would be to choose  $\sigma_0$  values corresponding to randomly generated values of the cumulative probability satisfying the above listed requirements. More important, probably, would be to proceed with families of clutter distributions characteristic of specific locales, as discussed below.

### C. USE OF LOCAL CLUTTER DISTRIBUTIONS

Clutter reflectivity distributions depend on the frequency, polarization, and spatial resolution of the radar, as well as environmental features such as terrain type, grazing angle, man-made features and vegetation cover. In order to obtain a single graph that indicates the characteristic vulnerability of a particular helicopter type to a given air defense system aggregating all the environmental features to obtain a single distribution for the purposes of analysis is acceptable. In the context of a detailed war game used for supporting acquisition decisions or training commanders, however, our full knowledge of the phenomenology should be incorporated.

There is still a shortage of data for ground-based radars. The following general considerations seem likely to apply, and can be used to outline future work. First, the broad distributions observed in nature (Fig. 3) are likely to be made up of several

distributions, for different terrain types, with different medians. These "fundamental" distributions are probably much less broad individually than is their sum. Finally, the median value (and possibly also the width) for a given terrain type will depend on the grazing angle between the radar transmitter and the illuminated ground patch. Given the knowledge of how the global distributions break down by terrain type, etc., the procedures outlined in Section IV.B could be extended.

For map-based war games using Defense Mapping Agency (DMA) data bases the needed information on terrain and grazing angle is available. To the extent that these features matter, they should be incorporated correctly in operational effectiveness studies and training exercises. A much more ambitious program would be to use these considerations in mission planning and execution (Ref. 4).

#### **D. SUMMARY**

The most important improvement in clutter modeling is also the easiest to implement in current models: If the terrain is masked set the clutter to zero.

To incorporate the broad range of clutter reflectivity, values using a single global distribution is the simplest next step. A better approach would be to incorporate what is known of the phenomenology of clutter reflectivity. Low-flying aircraft exploiting terrain will not find themselves exposed over random patches of terrain. Neglecting possible correlations between exposure points and clutter values could be very misleading in war games used for training or for analysis.

## V. EXPECTED IMPACT ON STUDIES

In this chapter, we will anticipate the effects the model improvements outlined earlier are likely to have.

Terrain-specific combat models must include line of sight to both targets and clutter. To our knowledge, line of sight to targets is generally treated, but some models assume clutter returns are always present, even if the ground is masked. Modeling techniques that account for masking of clutter will show enhanced effectiveness of air defense radars and reduced effectiveness of low-flying aircraft. Radars with such high clutter suppression or helicopters with such large signature that clutter never affords protection will be exceptions to this rule. An important question is "how often will helicopters be exposed while the ground under them is masked?" This can only be addressed in the context of exercise or war games focusing on specific missions and terrain.

Including the width of clutter distributions in the engagement of transiting targets leads to a probability of detection that limits very slowly to unity at short ranges. The timelines for engagement at short ranges will increase, but the possibility of engagement at long ranges may be slightly increased. Clutter will afford protection some of the time, even at very short range. This is a common experience in the field, but is often not reflected in combat models.

Modeling of hovering helicopters is still at a rudimentary level. We believe that the primary impact of the approach outlined here will be to change the modeled timelines for engagement. The changes are difficult to predict. It is likely that the distribution of timelines will be very broad, and cannot be modeled with a single delay.

## REFERENCES

1. D.A. Sparrow, I. Kay, R.R. Legault, J. Ralston, and W. Shafter, *Active and Passive Aids to Survivability for LHX (U)*, Institute for Defense Analyses, IDA Paper P-2285, January 1990 (SECRET/NOFORN/WNINTEL).
2. Merrill I. Skolnik, *Radar Handbook*, McGraw-Hill, 1970.
3. Fred E. Nathanson, *Radar Design Principles*, McGraw-Hill, 1969.
4. This possibility was brought to our attention by Maj. Gary Davis (USA).

## **APPENDIX**

### **TIMELINES AND M-OUT-OF-N RULES**

## APPENDIX

### TIMELINES AND M-OUT-OF-N RULES

Rules that impose constraints on whether or not to declare a detection based on a sequence of events are common in signal processing, both human and electronic. The earliest use of an m-out-of-n rule of which we are aware was descriptive: During World War II radar operators did not generally declare a target with a single scan, but waited for confirmation on one of the next two scans. This behavior was described as following a 2-out-of-3 rule for detections--a blip was required two times in three scans.

Beyond modeling human behavior such constraints are often imposed electronically, for example through requiring 20-out-of-30 pulses per scan, or 2-out-of-4 available doppler bins before a declaration is made. Electronic tests operating on a single scan affect the single scan probability of detection. The cumulative probabilities are straightforward to calculate from the underlying probabilities.

On the other hand, m-out-of-n rules applied to sequential scans alter the cumulative probability timelines in ways that can be difficult to calculate. In this appendix we outline the general algorithm, in the form of a recursion relationship, for computing how detection probability accumulates under an m-out-of-n rule. This solution is illustrated for the case of 2-out-of-3. A general solution is presented for the case of m-in-a-row. Cumulative probability and individual probability versus number of scans are computed for several values of  $P_D$  for the 2-out-of-3 rule. Several cases of how cumulative probabilities depend on range through the decrease of the single-scan probabilities with range are also presented.

#### A. GENERAL SOLUTION

A straightforward description of an algorithm to compute the m-out-of-n probabilities follows. First, one defines a vector with elements given by the probabilities that after j steps the m-out-of-n rule has not been satisfied, and the final  $n - 1$  elements in the string of hits and misses is specified by the set  $(\alpha)$ . One final element is the probability that a detection has occurred at or before this step:

$$V_j = [ (P_{ND}((\alpha), j)), P_D(j)]$$

The next step is to define a matrix,  $M$ , such that

$$V_{j+1} = M V_j .$$

One then defines by hand the  $V_j$  for  $j < n$ , and uses the matrix to iterate. In fact, since each element of  $V_j$  can only feed forward to two different elements of  $V_{j+1}$ , most elements of the matrix are zero. Since it is relatively easy to trace probabilities from step to step, this procedure is easily implemented on a spread sheet.

We may consider as an example the 2-out-of-3 rule. In this case, if we define  $P$  = the probability of a hit, and  $Q = 1 - P$  the probability of a miss, we have

$$V_2 = (QQ, PQ, QP, PP) ,$$

with the probability of satisfying the 2-out-of-3 rule given by the last element,

$$P_D = P^2 .$$

The matrix that propagates the vector is given by

$$M = \begin{bmatrix} Q & Q & 0 & 0 \\ 0 & 0 & Q & 0 \\ P & 0 & 0 & 0 \\ 0 & P & P & 1 \end{bmatrix} .$$

It is simple to check for the next step that this process is correct:

$$V_3 = M V_2 = (Q^3 + PQ^2, PQ^2, Q^2P, 2P^2Q + P^2) .$$

In this simple case a recursion relation can be found for the total  $P_{ND}$ ,

$$P_{ND}(j) = Q P_{ND}(j-1) + PQ^2 P_{ND}(j-3),$$

or for the incremental probability of detection  $\Delta P_D$ ,

$$\Delta P_D(j) = P_D(j-1) - P^2Q^2 P_D(j-4) - P^2Q^3 P_D(j-5).$$

These two expressions are consistent.

## B. M-IN-A-ROW

It is fortunate that large values of  $m$  and  $n$  normally are not used. The above outlined approach quickly becomes cumbersome for large values of  $n$ , provided that  $m \neq n$ . For  $m = n$ , a simple expression for a recursion relation can be derived following an approach similar to that outlined above. For  $m$ -in-a-row, however, the elements of the



vector are strings which have never had m-in-a-row hits, the first element ending in a miss, then a miss followed by a single hit, a miss followed by 2 hits..., a miss followed by m - 1 hits, and finally a string which has satisfied m-in-a-row at some time in its history:

$$V(P_{ND}(\dots 0), P_{ND}(\dots 01), P_{ND}(\dots 011), \dots P_D).$$

This results in the general expression for m-in-a-row

$$P_{ND}(j) = 1 \quad 0 \leq j < m$$

$$P_{ND}(j) = Q \sum_{i=1}^m P^{i-1} P_{ND}(j-i) \quad j \geq m.$$

This can be checked by evaluating  $P_{ND}$  for the first non-trivial case,  $j = m$ , which gives:

$$P_{ND}(j) = Q \sum_{i=1}^m P^{i-1}$$

$$= Q \frac{1 - P^m}{1 - P} = 1 - P^m.$$

Finally, we note that the  $P_{ND}(j)$  exhibit an exponential behavior for large  $j$ . In other words, in the limit as  $j$  becomes infinite  $P_{ND}(j+1)/P_{ND}(j)$  becomes a constant. This is trivially correct for  $m = 1$ , and appears to be true for all  $m$  and  $n$ . However, we have not found a way to exploit this to simplify analyses.

### C. SAMPLE CALCULATIONS USING A 2-OUT-OF-3 RULE

In general for m-out-of-(2m - 1) detection rules a single-scan probability of 50 percent accumulates to 50 percent after (2m - 1) scans. Hence, for single-scan probabilities greater than 50 percent, cumulative probabilities tend toward unity very fast. In Fig. A-1 we show how probabilities accumulate for low single-scan  $P_D$ . For moderate scan numbers, the cumulative probability is most sensitive to changes in the single-scan probability in the 0.1 to 0.2 range.

The probability per scan is plotted in Fig. A-2 for three values of  $P_D$ . We note first that for aircraft attrition rates of a few percent are generally considered unacceptably high. Even low per scan probabilities satisfy the 2-out-of-3 rule on the second or third scans at rates of a few percent. The cumulative probability derived from a 2-out-of-3 rule always peaks on either the second or third scan, depending on whether the single-scan probability is greater or less than 50 percent.

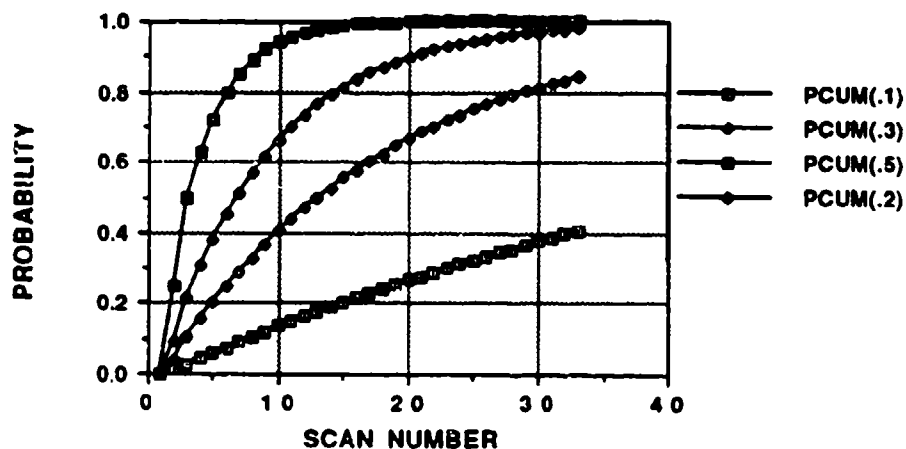


Figure A-1. Cumulative Probability for 2/3 Rule

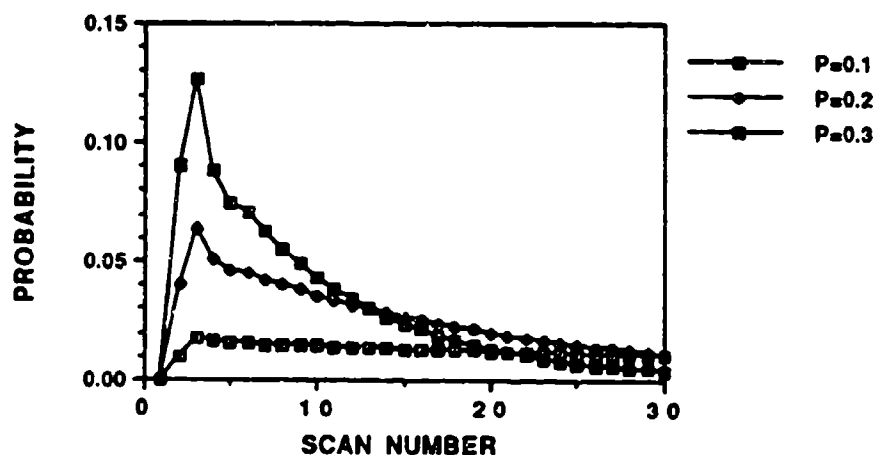


Figure A-2. Probability per Scan

We wish to examine what happens with these cumulative probabilities when viewed as a function of range for a fixed scan number rather than as a function of scan number for a fixed single-scan  $P_D$ . We use the new model for noise and clutter together, requiring both that S/C be greater than 13 dB, and that S/N lead to detection. This procedure generates the single-scan probability of detection indicated by the solid squares in Fig. A-3, and is repeated in Fig. A-4 here as open circles. At each range, this single-scan probability is used to generate a cumulative probability after 3, 5, and 10 scans. We see as expected that the probability accumulates fairly fast if the single-scan  $P_D$  is above 0.5. In other words, as the number of scans increase the cumulative probability curve gets steeper at the edge.

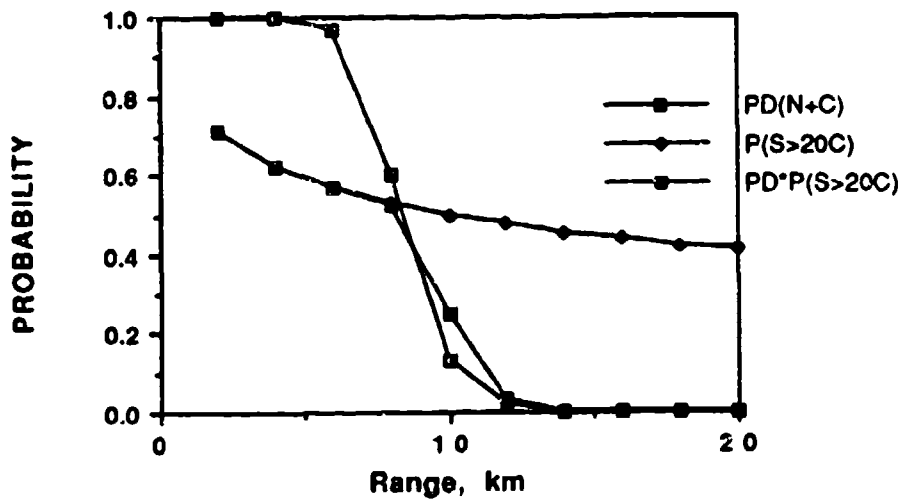


Figure A-3. Clutter-Limited Detection

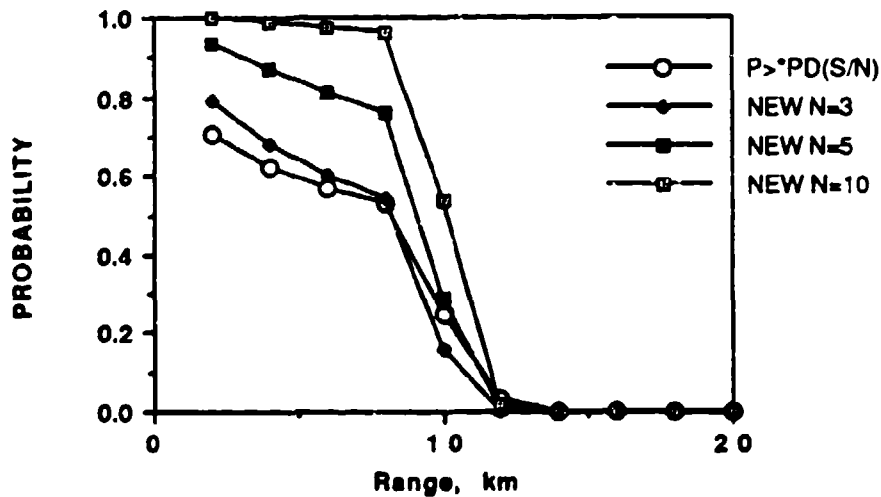


Figure A-4. Single-Scan and Cumulative  $P_D$ s

A quick look at Fig. A-3 suggests that using  $P_D(N + C)$  models the radar much more favorably than  $P_D(N) \cdot P(S > 20C)$ , since there is only a small region at fairly long range where the latter is superior. However, timelines as well as range enter into this consideration. It may be the case that probability of detection after a number of scans is a better measure of operational effectiveness than single-scan probability of detection. Figure A-5 presents the difference in cumulative probability after 3, 5, or 10 scans for the two single-scan probabilities from Fig. A-3. This figure reveals that for cumulative probability

after many scans, performance that is slightly better in the low single-scan  $P_D$  regime is more important than performance much better in the high  $P_D$  regime.

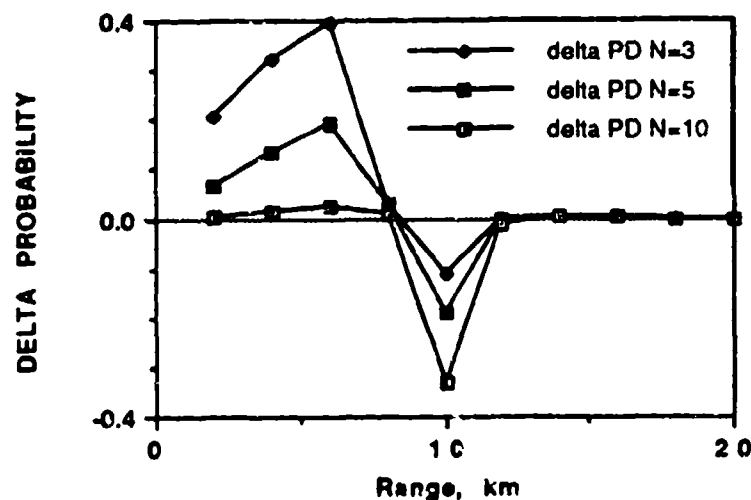


Figure A-5. Delta  $P_D$  Versus Range for Three Scan Numbers

Finally, we compare how the same fundamental probabilities accumulate for moving and stationary targets: the key difference is that the clutter does not change under a stationary helicopter.\* In other words, the probability that  $S > 20C$  cannot be allowed to accumulate to unity, since the clutter is whatever it is near the helicopter. In Fig. A-6, a given single-scan probability based on a product of noise and clutter probabilities is accumulated. For the solid diamonds, the entire probability is used, corresponding to significant scan-to-scan variation in the background clutter. For the solid squares, only the probability that noise limits detection is allowed to decay away, corresponding to a target in a fixed clutter cell. The differences are striking in the region where clutter dominates the single-scan  $P_D$ .

\* The formal assumption that the fundamental scan-to-scan probabilities is the same is critical here; clearly hovering and transmitting helicopters have very different single-scan detection probabilities in most cases.

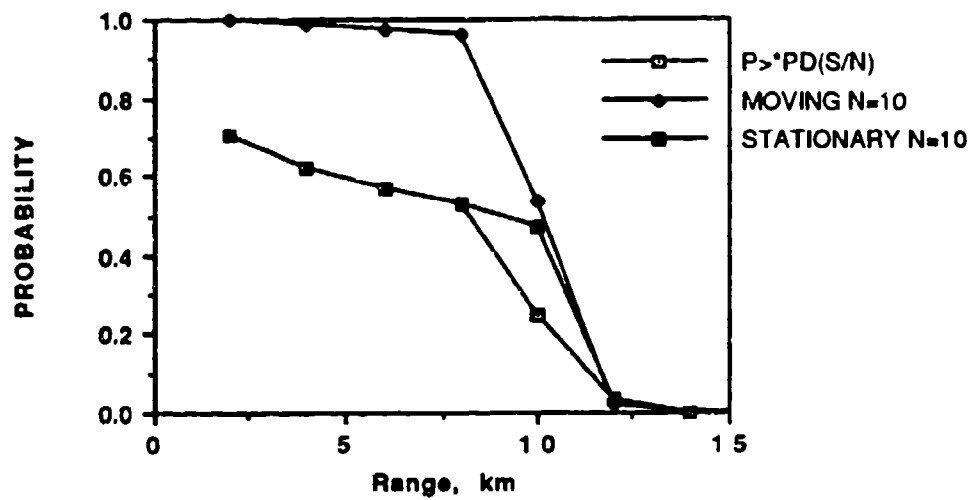


Figure A-6. Cumulative Probabilities for Two Cases